Subgame-perfect Cooperative Agreements in a Dynamic Game of Climate Change*

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Abstract

We model climate change as a dynamic game and prove existence of a *unique* subgame-perfect Nash equilibrium (SPNE) that is also Markov perfect. We interpret this unique SPNE as the business-as-usual (BAU) equilibrium and show that if the countries are not sufficiently symmetric then the familiar trigger strategy equilibria may not be Pareto improvements over the BAU equilibrium and may even lack efficiency properties. We then motivate and introduce a subgame-perfect cooperative agreement as an improvement over the BAU equilibrium in the sense that every country or coalition of countries is better off in *every* subgame, irrespective of the extent of heterogeneity of the countries. We characterize subgame-perfect cooperative agreements and identify sufficient conditions for their existence. We show that (direct or indirect) transfers between countries to balance the costs and benefits of controlling climate change are a necessity and not a matter of approach.

Keywords: Climate change, dynamic game, subgame-perfect transfers, trigger strategies.

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1. Introduction

In this paper, we model climate change as a dynamic game in discrete time in which each country chooses, in each period, its level of economic activity which generates benefits for the country as well as emissions that add to the existing stock of greenhouse gases (GHG), which causes climate change and negatively affects the welfare of all countries in the current and future periods. The benefit of each country in each period depends nonlinearly on its emissions – to reflect decreasing marginal abatement costs in emissions levels – while damages from climate change of each country depend nonlinearly or linearly on the GHG stock. The GHG stock evolves over time through additions due to emissions and depletions due to natural decay.

We identify a time-profile of emissions (and, therefore, of the GHG stock) that leads to an inter-temporally efficient/optimal outcome and show that it is unique. This means efficiency, i.e. optimal control of climate change, cannot be achieved unless the countries emit according to this unique efficient emissions time-profile. However, the costs and benefits of doing so differ over time and across countries, unless they are all identical. In fact, if the countries are not sufficiently symmetric, then their individual costs and benefits of controlling climate change cannot be balanced by just redistributing the total emissions in the unique efficient emissions time-profile among the countries.[1] Thus, only transfers between countries can balance the individual costs and benefits of controlling climate change and induce them to emit according to the unique efficient emissions time-profile. In other words, if the countries, as in reality, are sovereign and highly heterogeneous, then transfers to balance each country's costs and benefits of controlling

[1] Balancing costs and benefits here means that the benefit from controlling climate change to each country is not less than the cost of reducing its emissions.

climate change are a necessity and not a matter of approach.[2] Example 1 with two-countries and two periods below illustrates this fact. Additionally, the transfers should be such that they balance the costs and benefits of controlling climate change not only in the current state (i.e. the level of the GHG stock) but also in *every* state that may occur in future. We motivate and interpret efficient time-profile of emissions and streams of transfers with this property as subgame-perfect cooperative agreements. To be precise, an agreement is subgame-perfect if no country or coalition of countries wants to withdraw from it in *any* subgame. We identify sufficient conditions for the existence of such agreements and characterize them.

Our model with non-linear benefit and non-linear or linear damage functions and many heterogeneous countries builds on the existing dynamic models. The two papers closest to the current one are Dockner et al. (1996) and Dutta and Radner (2009). However, unlike Dockner et al. (1996), we do not assume identical and constant marginal abatement costs, since there is compelling evidence that in reality the marginal abatement costs are decreasing in emissions levels (see e.g. Nordhaus and Boyer, 2000) and differ significantly across the countries (see e.g. Ellerman and Decaux, 1998). Thus we assume instead decreasing marginal abatement costs that may differ across countries to any extent. As will be shown below, decreasing marginal

[2] However, many preceding papers, for reasons of tractability, get around the necessity of transfers by assuming sufficiently symmetriccountries such that the costs and benefits of controlling climate change can be balanced without transfers. In contrast, transfers are implicit in Coase's (1960) classical solution for tackling externalities which requires direct transfers between the parties involved for reducing an externality. As shown in Ellerman and Decaux (1998) and Chander (2003), transfers were also implicit in the Kyoto Protocol via the Clean Development Mechanism and the assignment of emission quotas that could be traded on an international market. The Montreal Protocol, which has been hailed as an example of successful international cooperation, explicitly requires transfers, though not in as large amounts as those implicit in the Kyoto Protocol.

abatement costs, unlike identical and constant marginal abatement costs, imply a *unique* subgame-perfect Nash equilibrium (SPNE) that is also Markov perfect.[3]

Dutta and Radner (2009) introduce and characterize "global Pareto optimal" (GPO) consumption time-profiles and show that they can be supported as history dependent subgame-perfect equilibrium outcomes through the use of trigger strategies. However, as will be shown below, the set of GPO consumption time-profiles includes just one consumption time-profile that is Pareto efficient – the rest are not. This means that, unlike a subgame-perfect cooperative agreement proposed in this paper, the GPO consumption time-profiles that can be supported as trigger strategy equilibria may not be Pareto efficient. In fact, they may also not be Pareto improvements over the unique SPNE (referred to as the "business as usual" equilibrium in Dutta and Radner, 2009) in the sense that some country may be worse off compared to the "business as usual" outcome, unless the countries are sufficiently symmetric. This is because, unlike subgame-perfect cooperative agreements, GPO consumption time-profiles, by definition, rule out transfers. But if the countries are not sufficiently symmetric then, as argued above, the costs and benefits of controlling climate change cannot be balanced without transfers between the countries. In contrast, the subgame-perfect cooperative agreements balance the costs and benefits of controlling climate change in each subgame irrespective of the extent of heterogeneity of the countries. A similar remark applies to Dockner et al. (1996) who also rule out transfers and impose limits on the extent of heterogeneity, besides assuming just two countries. Though the

[3] As will be seen from the proof of Theorem 1 below, identical and constant marginal abatement cost functions can give rise to multiple SPNE and, thus, multiple Markov perfect equilibria. They also rule out possibility of any gains from international trade in emissions in contradiction to the Kyoto Protocol that proposed international trade in emissions to significantly reduce the costs of meeting the obligations under the Protocol.

trigger strategy equilibria in their paper are efficient, both countries may not be individually better off compared to their Markov perfect equilibrium (MPE) payoffs, especially if the countries are not sufficiently symmetric.

Other related papers include Rubio and Ulph (2007) who study the impact of changes in the GHG stock on the internal-external stability of a coalition in a dynamic game.[4] As in the present paper, they also assume that when a coalition forms the remaining players follow their individually best reply strategies. But unlike the present paper, they assume symmetric countries. There is also a fairly small but important literature on dynamic games for efficient public good provision including the papers by Marx and Matthews (2000) and Harstad (2012).[5] Like Dockner et al. (1996), Marx and Matthews (2000) make assumptions that are equivalent to assuming constant and identical marginal abatement costs and thus they too face the problem of choosing among alternative MPEs.[6] Marx and Matthews, (2000) prove existence of efficient Bayesian equilibria sustained by trigger strategies that impose maximal possible punishment. This punishment, as will be seen below, is conceptually harsher than adopting individually best reply strategies assumed in the present paper. In contrast, Harstad (2012) studies incentives to invest in green technologies that aid in reducing emissions. Countries face no threat of punishment and can write contracts that commit them to a time-profile of emissions. Though innovative and insightful, Harstad, for reasons of tractability, abstracts from the fact of highly asymmetric

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[4] A coalition is internally stable if no member of the coalition can be better off by leaving the coalition – assuming that the coalition left behind will remain stable – and externally stable if no non-member can be better off by joining the coalition – assuming that the expanded coalition will be stable.

[5] Also, see Chander (1993) for a differential game model of public good provision.

[6] Marx and Matthew (2000: p.348) propose to extend their analysis to the case in which the players are more heterogeneous in future research.

countries. As a result, both international trade in emissions and transfers are missing from his analysis. He also does not consider coalitional behavior and possibility of a country opting out of an agreement.

As in the static model of climate change in Chander and Tulkens (1997), we also assume in the dynamic model that each deviating coalition believes that upon its withdrawal from an agreement the other countries will react by choosing their *individually* best reply strategies. We interpret this as imposing a punishment on the deviating coalition that is similar to that in Dockner et al. (1996) and Dutta and Radner (2009), except that in this paper it is shown to be sufficient to deter deviations not only by single countries but also by coalitions of many countries. This is significant because some countries often respond to proposals for agreements to control climate change as a group. E.g., the developing and developed countries often react to proposals for agreements as separate blocs. Thus, we require a subgame-perfect cooperative agreement to be robust not only against withdrawals by single countries but also by coalitions of many countries. This leads to a more exclusive notion of a subgame-perfect cooperative agreement and thus to more restrictive sufficient conditions for its existence.

We begin introducing subgame-perfect cooperative agreements by showing that the dynamic game with any finite number of not necessarily symmetric countries admits a *unique* subgame-perfect Nash equilibrium (SPNE) if the benefit functions are *strictly* concave and quadratic (i.e. the marginal abatement costs are strictly decreasing in emissions levels) and the damage functions are quadratic or linear (i.e. the marginal damages from climate change are nondecreasing in the GHG stock levels).[7] This SPNE, as will be shown, is a Markov perfect equilibrium (MPE), i.e., the SPNE strategies are functions of the current state variable.[8] The proof uses the method of backward induction and is quite constructive. As in the preceding studies, the unique SPNE is not efficient and the equilibrium emissions are typically different from those in the unique efficient emissions time-profile. This motivates us to propose a subgame-perfect cooperative agreement as a Pareto improvement over the unique SPNE in the sense that the agreement would not only optimally control climate change but also every country or coalition of countries would be better off in *every* subgame and thus no country or coalition of countries will have incentives to withdraw from the agreement in any subgame.[9]

For the sake of a simple and transparent analysis, this paper focuses mostly on a dynamic game with finite horizon, but also discusses extensions to the infinite horizon version of the game. In fact, as will be seen below, most of our analysis can be extended to the infinite horizon version.

The contents of this paper are as follows: Section 2 describes the dynamic model and compares Pareto efficient and "global Pareto optimal" consumption time-profiles. Section 3 formally states the dynamic game and proves existence of a unique SPNE and shows that it is

[7] Germain et al. (2003: last line of p.82) also claim existence and uniqueness of a closed-loop Nash equilibrium in a similar dynamic game under more general conditions. But they do not prove the same and, in fact, their claim is false and based on the wrong belief that the conditions for the existence of a closed-loop Nash equilibrium in a dynamic game are the same as the conditions for the existence of a Nash equilibrium in a strategic game.

[8] In contrast, the dynamic game in Dockner et al. (1996) admits multiple Markov perfect equilibria. As will be made clear below, this is because in their model the marginal abatement costs are assumed to be identical and constant across countries.

[9] Agreements involving transfers have been considered previously in a similar dynamic model by Germain et al. (2003), but those agreements have not been shown to be improvements over the SPNE and satisfy subgame perfection.

also Markov perfect. Section 4 motivates and introduces the concept of a subgame-perfect cooperative agreement and proves existence in the case of linear damage functions. Section 5 draws the conclusion. All proofs are gathered in the Appendix to the paper.

2. The dynamic model

There are n countries, indexed by $i = 1, \dots, n$, and $N = \{1, \dots, n\}$ is the set of all countries. Time is discrete, indexed by $t = 1, \dots, T$, where T is finite but may approach infinity. The variables $x_{it} \geq 0$ and $y_{it} \geq 0$ denote the consumption and production (resp.) of a composite private good of country i in period t; x_{it} and y_{it} may differ because transfers between countries are permitted. The variables $e_{it} \geq 0$ and $z_t \geq 0$ denote (resp.) the amount of GHG emitted by country i and the GHG stock in period t. While x_{it}, y_{it}, and e_{it} are flow variables, z_t is a stock variable which evolves overtime according to Eq. (2) below. The output of the private good and emissions of each country i are related according to the equation $y_{it} = g_i(e_{it})$ where $g_i(e_{it})$ is the benefit function. Each country i suffers damages from climate change and derives utility from the private good consumption in each period t according to the (utility) function $v_i(x_{it}, z_t) = x_{it} - d_i(z_t)$, where x_{it} is the private good consumption and $d_i(z_t)$ is the damage function. Thus, utility is transferable unit for unit (i.e. each country can increase the payoff of any other country by one unit at the cost of exactly one unit to itself) and the model is similar to the classical model with one private and one public good and quasi-linear utility functions except that it is dynamic, the endowments of the private good are not exogenously fixed, and the public good is a public bad. Assuming a more general utility function will complicate the analysis, but will not lead to additional insights. For this reason, as in most preceding papers, we assume quasi-linear utility functions.

We assume that the benefit function $g_i(e_{it})$ of each country i is strictly increasing and strictly concave, and the damage function, $d_i(z_t)$ is strictly increasing and convex or linear, i.e., $g_i'(e_{it}) > 0, g_i''(e_{it}) < 0, d_i'(z_t) > 0,$ and $d_i''(z_t) \geq 0.$ Depending on the context, we interpret the derivative $g_i'(e_{it})$ as the marginal abatement cost or the marginal benefit of emissions, e_{it}, of country i, and the derivative $d_i'(z_t)$ as the marginal damages of country i due to the GHG stock z_t. We also assume that for all $z \geq 0$ and each country i, there exists an $e^0 > 0$ such that $g_i'(e^0) \leq d_i'(e^0 + z)$ and $\lim_{e_i \to 0} g_i'(e_i) = \infty > \sum_{j \in N} d_i'(z)$. This assumption means that for all levels of the GHG stock z, the marginal benefit of emissions for each country i is smaller (resp. larger) than its own marginal damages for large (resp. small) enough emissions. As will be seen, the assumption ensures that each utility maximizing country i will choose its emissions e_{it} in period t such that $0 < e_{it} \leq e^0, t = 1, \ldots, T$.

Given an initial GHG stock $z_0 \geq 0$, a time-profile of consumption $(x_{1t}, \ldots, x_{nt}; z_t)_{t=1}^T$ is *feasible* if there exists a time-profile of emissions $(e_{1t,} \ldots, e_{nt})_{t=1}^T$ such that

$$\sum_{i=1}^n x_{it} = \sum_{i=1}^n g_i(e_{it}) = \sum_{i=1}^n y_{it} \tag{1}$$

and
$$z_t = (1 - \delta)z_{t-1} + \sum_{i=1}^n e_{it}, t = 1, \ldots, T. \tag{2}$$

Here $0 \leq \delta \leq 1$ is the natural rate of decay of the GHG stock. To minimize notation, we assume henceforth that the initial GHG stock is fixed at $z_0 = 0$.

Since Eq. (1) does *not* require $x_{it} = g_i(e_{it}) = y_{it}$ for each i, it permits transfers of the private good between countries in each period t, but not across the periods. Given the quasi-linearity of the utility functions $v_i(x_{it}, z_t)$, the latter is not really an assumption as there is no gain from postponing consumption and there is no possibility of borrowing against future consumption for the world as a whole. Each feasible consumption time-profile $(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^{T}$ uniquely generates an aggregate utility $\sum_{t=1}^{T} \beta^{t-1} v_i(x_{it}, z_t) = \sum_{t=1}^{T} \beta^{t-1} [x_{it} - d_i(z_t)]$ for each country i where $0 < \beta \leq 1$ is the constant discount factor, assumed, for the sake of simplicity, to be the same for all countries.

In the optimal control literature, the GHG emissions $(e_{it})_{t=1}^{T}, i = 1, \dots, n$, are called *control variables* and the resulting GHG stocks $z_{t-1}, t = 1, \dots, T$, the *state variables*. While the latter are not strategies in the dynamic game introduced below, they are generated by the former and appear in the payoff functions of the countries. In fact, they have the same role as decision nodes in a dynamic game.

2.1 *Efficient emissions and consumption time-profiles*

Since climate agreements aim at maximizing social welfare, we define and characterize efficient consumption time-profiles and compare them with "global Pareto optimal" consumption time-profiles.

Definition 1 Given an initial stock $z_0 \geq 0$, a feasible consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^{T}$ is (Pareto) efficient if there is no other feasible consumption time-profile

$(x'_{1t}, \dots, x'_{nt}; z'_t)_{t=1}^T$ such that $\sum_{t=1}^T \beta^{t-1} v_i(x'_{it}, z'_t) \geq \sum_{t=1}^T \beta^{t-1} v_i(x^*_{it}, z^*_t)$ for each $i = 1, \dots, n$ with strict inequality for at least one i.

Since $v_i(x_{it}, z_t) = x_{it} - d_i(z_t)$, i.e. utility is transferable (unit for unit) between the countries, Definition 1 implies that a consumption time-profile $(x^*_{1t}, \dots, x^*_{nt}; z^*_t)_{t=1}^T$ is efficient *if and only if* it is a solution of the optimization problem $max_{(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^T} \sum_{i=1}^n \sum_{t=1}^T \beta^{t-1} [x_{it} - d_i(z_t)]$ subject to (1) and (2). After switching the summation signs and substituting from (1), this optimization problem is equivalent to $max_{(e_{1t}, \dots, e_{nt}; z_t)_{t=1}^T} W = \sum_{t=1}^T \beta^{t-1} \sum_{i=1}^n [g_i(e_{it}) - d_i(z_t)]$ subject to (2). It is easy to show that the solution to this optimization problem must satisfy

$$g'_i(e_{it}) = \sum_{\tau=t}^T [\beta(1-\delta)]^{\tau-t} \sum_{j \in N} d'_j(z_\tau), i = 1, \dots, n, t = 1, \dots, T. \qquad (3)$$

Equalities (3) embody two distinct notions of efficiency: (a) the marginal abatement costs of all countries must be equal in each period t and (b) the marginal abatement cost of each country i in each period t must be equal to the sum of discounted marginal damages of all countries that will be avoided, over the remaining time horizon $T - t + 1$, if the emissions of country i were reduced by one unit in period t. While (a) is sometimes referred to as the *equi-marginal cost* principle (e.g. Kolstad, 2000) or cost efficiency, (b) is the dynamic version of the well-known Lindahl-Samuelson condition for efficient provision of a public good. For this reason, we shall refer to (3) as the efficiency condition. Since W is a strictly concave function, a consumption time-profile $(x^*_{1t}, \dots, x^*_{nt}; z^*_t)_{t=1}^T$ is efficient if and only if it satisfies equalities (1) – (3). Thus, the solutions to equalities (1) – (3) describe the set of *all* efficient consumption time-profiles. The following lemma characterizes this set.

Lemma 1 All efficient consumption time-profiles $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ are generated by the same emissions time-profile $(e_{1t}^* \dots, e_{nt}^*)_{t=1}^T$.

The proof of this lemma follows from the fact that equalities (2) and (3) admit a unique solution. See the Appendix to the paper for details. In view of this result, we shall refer to $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ as the unique efficient emissions time-profile and to $(g_1(e_{1t}^*), \dots, g_n(e_{nt}^*); z_t^*)_{t=1}^T$ as the unique efficient consumption time-profile *without* transfers.

The lemma has an important implication in that it implies that efficiency cannot be achieved unless all countries emit according to the unique efficient emissions time-profile $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ and for each efficient consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$, we have $\sum_{i \in N} x_{it}^* = \sum_{i \in N} g_i(e_{it}^*)$, $t = 1, \dots, T$. Thus, the efficient consumption time-profiles differ from each other *only* in terms of the individual private good consumption: the time-profile of emissions and, thus, the GHG stock is the same in *all* efficient consumption time-profiles. All but the unique efficient consumption time-profile without transfers, $(g_1(e_{1t}^*), \dots, g_n(e_{nt}^*); z_t^*)_{t=1}^T$, require transfers of the private good between the countries.

Before ending this section we note that each efficient consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ is equivalent to a time-profile $(e_{1t}^0, \dots, e_{nt}^0)_{t=1}^T$ of emission rights that are tradeable on a competitive international market and such that $\sum_{i=1}^n e_{it}^0 = \sum_{i=1}^n e_{it}^*$, where

$(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ is the unique efficient emissions time-profile, and each country i will choose to buy/sell $e_{it}^0 - e_{it}^*$ of rights at the competitive price in period t and emit e_{it}^*. It is worth noting that the right e_{it}^0 can be negative meaning that country i must buy emissions rights that are more than the amount e_{it}^* it can emit under the unique efficient emissions time-profile. It is easily seen that the market for emission rights clears in each period t if the international price is $p_t^* = \sum_{\tau=t}^T [\beta(1-\delta)]^{\tau-t} \sum_{j \in N} d_j'(z_\tau^*)$ and each country chooses its emissions to maximize its payoff $g_i(e_{it}) + p_t^*(e_{it}^0 - e_{it})$.[10]

2.2 *Infinite time horizon and efficiency*

If the time horizon $T = \infty, 0 < \beta(1-\delta) < 1$, and each marginal damage function $d_i'(z_t)$ is bounded above equalities (2) and (3) still hold . This is indeed the case if the damage functions are linear, i.e., $d_i(z) = c_i z, i = 1, \dots, n,$ and thus $d_i'(z) = c_i, i = 1, \dots, n,$ and the efficiency conditions (3) then take a simple form in that the efficient time-profile of emissions is given simply by the equalities

$$g_i'(e_{it}^*) = \frac{\sum_{j=1}^n c_j}{1-\beta(1-\delta)}, i = 1, \dots, n, t = 1, \dots \quad . \tag{4}$$

Dutta and Radner (2009) assume linear damage functions and introduce the notion of a "global Pareto optimal" (GPO) consumption time-profile. Given a (exogenous) profile of welfare weights $(\alpha_1, \dots, \alpha_n)$ with $\alpha_i > 0, i = 1, \dots, n, \sum_{i=1}^n \alpha_i = 1$, $(\hat{x}_{1t}, \dots, \hat{x}_{nt}; \hat{z}_t)_{t=1}^\infty$ is a GPO consumption time-profile if $\hat{x}_{it} = g_i(\hat{e}_{it}), i = 1, \dots, n,$ and $(\hat{e}_{1t}, \dots, \hat{e}_{nt})_{t=1}^\infty$ is the solution of the

[10] It is assumed here that emission rights for period t can be traded only in period t.

optimization problem $max_{(e_{1t},\dots,e_{nt};z_t)_{t=1}^{\infty}} W^{\alpha} = \sum_{i=1}^{n} \alpha_i \sum_{t=1}^{\infty} \beta^{t-1}[g_i(e_{it}) - d_i(z_t)]$ subject to (2). Dutta and Radner (2009: Theorem 1) show that the solution to this optimization problem is a unique emissions time-profile $\hat{e} = (\hat{e}_{1t}, \dots, \hat{e}_{nt})_{t=1}^{\infty}$ characterized by the equalities

$$\alpha_i g_i'(\hat{e}_{it}) = \frac{\sum_{j=1}^{n} \alpha_j c_j}{1 - \beta(1-\delta)}, i = 1, \dots, n, t = 1, \dots \tag{5}$$

Proposition 1 If the damage functions are linear, the GPO consumption time-profile corresponding to equal welfare weights, i.e. $\alpha_i = \alpha_j, i, j = 1, \dots, n,$ is efficient. No other GPO consumption time-profile is efficient.

The proof of this proposition follows from Lemma 1 that characterizes the set of all efficient consumption time-profiles and a comparison of equalities (5) with equalities (4) that characterize the unique efficient emissions time-profile for linear damage functions. Also notice that no GPO consumption time-profile except the unique *efficient* GPO consumption time-profile satisfies even cost efficiency (i.e. the equi-marginal cost principle), let alone efficiency.

We prove Proposition 1 only for linear damage functions because Dutta and Radner (2009) assume the damage functions to be linear. However, the GPO consumption time-profiles can be shown to be also not efficient for strictly convex damage functions. This is because unless the welfare weights are all equal, the optimization problem $max_{(e_{1t},\dots,e_{nt};z_t)_{t=1}^{\infty}} W^{\alpha}$ that characterizes the GPO consumptiom time-profiles then also differs from the optimization problem $max_{(e_{1t},\dots,e_{nt};z_t)_{t=1}^{T}} W$ that characterizes the efficient consumption time-profiles.

In contrast to Dutta and Radner (2009), Dockner et al. (1996) assume strictly convex damage functions, but identical and linear benefit functions and only two countries. It is easily seen that if the benefit functions, g_i, are identical and linear, then equations (2) and (3) admit infinitely many solutions because then the term on the left in (3), $g_i'(e_{it})$, is a constant and the same for all i. The set of these infinitely many solutions include efficient emissions time-profiles which are such that either *only* country 1 or 2 emits and produces and consumes the entire amount of the private good. Dockner et al. (1996) show that any of these infinitely many efficient emissions time-profiles can be implemented as a history dependent subgame-perfect equilibrium through the use of trigger strategies. However, they do not address the question of which of these infinitely many efficient emissions time-profiles may be actually selected for implementation. In fact, since these efficient emissions time-profiles have quite different welfare implications and, by assumption, transfers are ruled out, the countries may not agree to any efficient emission time-profile for implementation. Either country 1 or 2 may refuse to participate in the game unless they are both better off compared to their BAU equilibrium payoffs.

3. The dynamic game

Given an initial stock $z_0 \geq 0$ and time periods $T > 1$, Γ_{z_0} denotes the dynamic game in which

- $N = \{i = 1, 2, \ldots, n\}$ is the player set
- $E = E_1 \times E_2 \times \cdots \times E_n$, where $E_i = \{e_i \equiv (e_{it})_{t=1}^{T} : 0 \leq e_{it} \leq e^0\}$, is the set of all terminal histories

- $u = (u_1, \dots, u_n)$ is the profile of payoff functions such that for each terminal history $e \equiv (e_1, \dots, e_n) \equiv ((e_{1t})_{t=1}^{T}, \dots, (e_{nt})_{t=1}^{T}) \in E,\ u_i(e) = \sum_{t=1}^{T} \beta^{t-1}[g_i(e_{it}) - d_i\,(z_t)],$ where $z_t = (1-\delta)z_{t-1} + \sum_{j \in N} e_{jt}\,, t = 1, \dots, T.$

Let $\Gamma_{z_{t-1}}, z_{t-1} \geq 0, t \geq 1$, denote the dynamic game in which also the player set is N, the set of all terminal histories is $E_{1t} \times E_{2t} \times \cdots \times E_{nt}$, where $E_{it} = \{(e_{i\tau})_{\tau=t}^{T} : 0 \leq e_{i\tau} \leq e^0\}$, and the payoff of player i for each terminal history $e_t \equiv ((e_{1\tau})_{\tau=t}^{T}, \dots, (e_{n\tau})_{\tau=t}^{T}) \in E_{1t} \times E_{2t} \times \cdots \times E_{nt}$ is $u_i(e_t) = \sum_{\tau=t}^{T} \beta^{\tau-1}[g_i(e_{i\tau}) - d_i\,(z_\tau)],$ where $z_\tau = (1-\delta)z_{\tau-1} + \sum_{j \in N} e_{j\tau}, t = 1, \dots, T$. The game $\Gamma_{z_{t-1}}$has exactly the same structure as the original game Γ_{z_0} except that its origin is at z_{t-1} and the time horizon is shorter. The dynamic game $\Gamma_{z_{t-1}}, t \in \{1, \dots, T\}$, is a subgame of Γ_{z_0} if there exists a (non-terminal) history $((e_{1\tau})_{\tau=1}^{t-1}, \dots, (e_{n\tau})_{\tau=1}^{t-1})$ such that $z_{\tau-1} = (1-\delta)z_{\tau-2} + \sum_{j \in N} e_{j\tau-2}\,, \tau = 2, \dots, t$. However, notice that each subgame $\Gamma_{z_{t-1}}$depends only on z_{t-1} and not on the history before the game reaches the state z_{t-1}.

A *strategy* of player i is a function $e_i(z_{t-1}), 0 \leq e_i(z_{t-1}) \leq e^0, t = 1, \dots, T$, i.e., the emissions of player i in any period depend only on the current GHG stock. This type of strategies are called Markovian. More conventionally, a Markov strategy is a function $e_i(z, t)$, but to save on notation we simply define it by $e_i(z_{t-1})$. This should not cause any confusion.

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3.1 *The Subgame-perfect Nash equilibrium*

In order to decide whether to sign an agreement each country should be able to compare its payoffs when it does and does not sign the agreement. In this regard, we show that the dynamic game admits a unique subgame-perfect Nash equilibrium (SPNE) and interpret it as the situation that will prevail in the absence of an agreement, i.e., as the BAU equilibrium. The proof will also help us to make clear that the dynamic game may admit multiple SPNEs if, as in Dockner et al. (1996), the benefit functions are identical and linear and not strictly concave.

Theorem 1 The dynamic game Γ_{z_0} admits a unique subgame-perfect Nash equilibrium if the benefit functions $g_i, i = 1, \dots n$ are strictly concave, the damage functions $d_i, i = 1, \dots, n$ are strictly convex or linear, and the third derivatives $g_i''' = d_i''' = 0$, $i = 1, \dots n$, i.e., all benefit and damage functions are quadratic.

The proof of this theorem uses the method of backward induction and consists of the following steps: First prove that each subgame $\Gamma_{z_{T-1}}, z_{T-1} \geq 0$, in the last period T admits a unique SPNE $(e_{1T}(z_{T-1}), \dots, e_{nT}(z_{T-1}))$ which results in payoffs $q_i(z_{T-1}) \equiv g_i\big(e_{iT}(z_{T-1})\big) - d_i\big((1-\delta)z_{T-1} + \sum_{j \in N} e_{jT}(z_{T-1})\big), i = 1, \dots, n$. Then, using the assumptions regarding the functions g_i and d_i, prove that each $q_i(z_{T-1})$, $z_{T-1} \geq 0$, is a concave function and $q_i'''(z_{T-1}) = 0$ and note that the players' payoff functions in the reduced form of the subgame $\Gamma_{z_{T-2}}, z_{T-2} \geq 0$, in period $T-1$ are given by $g_i(e_{iT-1}) - [d_i\left((1-\delta)z_{T-2} + \sum_{j \in N} e_{jT-1}\right) - q_i\big((1-\delta)z_{T-2} + \sum_{j \in N} e_{jT-1}\big)]$, $i = 1, \dots, n$, and satisfy concavity in exactly the same way as do the payoff functions in the subgame $\Gamma_{z_{T-1}}, z_{T-1} \geq 0$, in period T. Thus, the reduced form of the subgame $\Gamma_{z_{T-2}}$ has exactly the same mathematical structure as the game $\Gamma_{z_{T-1}}$, as $q_i(z_{T-1}), i = 1, \dots, n$, is a non-increasing concave and quadratic function of z_{T-1}. Therefore, $\Gamma_{z_{T-2}}$ admits a unique SPNE and the SPNE payoffs $q_i(z_{T-2}), i = 1, \dots, n$, are similarly non-increasing concave and quadratic functions of z_{T-2}. Continuing in this manner, the backward induction leads to a unique SPNE of the dynamic game Γ_{z_0} .

Corollary 1.1 The subgame-perfect Nash equilibrium strategy of each player is linear and non-increasing in the state variable $z_{t-1}, t = 1, \dots, T$.

The corollary follows from equations (7) and (8) in the Appendix that show that the subgame-perfect equilibrium strategy $e_{it}(z_{t-1})$ of each player i satisfies $e'_{it}(z_{t-1}) \leq 0$ and $e''_{it}(z_{t-1}) = 0$. Thus, the SPNE is also Markov perfect.

Corollary 1.2 The unique subgame-perfect Nash equilibrium is not efficient.

If $\beta = 1$ and $\delta = 0$, then the efficiency conditions (3) implies $g'_i(e_{iT}) = \sum_{j \in N} d'_j(z_T)\,, i = 1, \dots, n$. Comparing them with (6) in the Appendix implies that the SPNE outcome is inefficient. Clearly, this is also true for $\beta \leq 1$ and $1 \geq \delta \geq 0$.

Corollary 1.3 If the benefit functions are identical and linear and the damage functions are strictly convex, the game admits infinitely many subgame-perfect Nash equilibria.

If the benefit functions are linear and identical, equations (6) in the Appendix admit infinitely many solutions such that only the sum total of emissions is uniquely determined, but not the individual emissions. In fact, in this case, equations (7) and (8) do not reveal much about the characteristics of the individual equilibrium strategies except in the case of symmetric countries.

Since a SPNE is a non-cooperative solution concept in which the *countries* maximize their own individual payoffs without any consideration whatsoever regarding the damages their actions inflict on the other countries, we shall refer to it as the *status quo*, i.e., the situation that will prevail in the absence of an agreement. For this reason, it is sometimes also referred to as the business-as-usual (BAU) equilibrium (see e.g. Dutta and Radner, 2009). Our assumptions regarding the benefit and damage functions, though weaker than those in the previous literature (since neither the countries are assumed to be symmetric nor the damage functions are assumed to be linear or identical), are sufficient for an analytical proof for the existence of a SPNE.[11]

We can also prove existence of yet another type of equilibria for the dynamic game, namely: equilibria in open loop strategies. However, this type of equilibria require high commitment on the part of the countries and thus are not appropriate – in the present context – for describing the equilibrium payoffs of countries in all possible states that may occur in a future period due to e.g. an action of a country that was not anticipated such as failing to follow its equilibrium strategy. However, in the case of linear damage functions, as can be shown, the unique SPNE is also an open-loop equilibrium.

3.2 *Infinite time horizon and the SPNE*

[11] Mäler and de Zeeuw (1998: Eq. 3) prove existence of a unique MPE in linear strategies, but for less general damage functions of the form $d_i(z) = c_i z^2, c_i > 0, i = 1, \ldots, n$. In contrast, Theorem 1 here holds for general quadratic damage functions of the form $d_i(z) = a_i + b_i z + c_i z^2$ where any of the parameters a_i, b_i, and c_i may be equal to zero for some countries, but not for others.

If the time horizon $T = \infty$, the existence of a SPNE or equivalently a MPE of the dynamic game Γ_{z_0} can be proved by showing that the functional equations $q_i(z) = g_i(e_{i1}(z)) - [d_i\left((1-\delta)z + \sum_{j\in N} e_{j1}(z)\right) - \beta q_i\left((1-\delta)z + \sum_{j\in N} e_{j1}(z)\right)], i \in N$, with $g_i''' = d_i''' = 0, i \in N$, admit a solution $q_i(z), i \in N$, such that each $q_i(z)$ is an increasing and concave function with $q_i''' = 0$, where $(e_{11}(z), \ldots, e_{n1}(z))$ is the Nash equilibrium of the strategic game with payoff functions given by $g_i(e_{i1}) - (d_i\left((1-\delta)z + \sum_{j\in N} e_{j1}\right) - \beta q_i((1-\delta)z + \sum_{j\in N} e_{j1}))$, $i \in N$. One can guess that the solution of the above functional equations is of the form $q_i(z) = (a_i z + b_i)^2$ and then find the values of the parameters a_i and b_i such that the conditions for the Nash equilibrium of the strategic game as well as the functional equations $q_i(z) = g_i(e_{i1}(z)) - \left[d_i\left((1-\delta)z + \sum_{j\in N} e_{j1}(z)\right) - \beta q_i\left((1-\delta)z + \sum_{j\in N} e_{j1}(z)\right)\right], i \in N$, are satisfied for all values of z.

This line of proof, however, does not prove that the MPE is unique. In fact, for a model with two symmetric countries and infinite time horizon, Dockner and Long (1993) show that the dynamic game may also admit a Markov perfect equilibrium in non-linear strategies which is better for both the countries and can even sustain an efficient consumption time-profile. However, this line of research still has to be developed for the asymmetric case with more than two countries, which is the model in this paper. As Dockner and Long (1993) note "If countries are highly asymmetrical, it would be difficult to agree on the selection of a given pair of strategies."

3.3 *The Necessity of transfers*

To further reinforce our claim that transfers between countries that are not sufficiently symmetric are necessary, we now return to the well-known model by Dutta and Radner (2009) and show that if the countries are not sufficiently symmetric, then the unique efficient GPO consumption time-profile is not a Pareto improvement over the SPNE, i.e., the BAU equilibrium.

Proposition 2 If the damage functions are linear and the countries are not sufficiently symmetric, the unique efficient GPO consumption time-profile is not a Pareto improvement over the BAU equilibrium.

Propositions 1 and 2 together show that if the countries are not sufficiently symmetric, the GPO consumption time-profiles that can be supported as the outcomes of history dependent subgame-perfect equilibria through the use of trigger strategies may be either inefficient or not a Pareto improvement over the SPNE . Typically countries with relatively low marginal damages will be worse off if efficiency requires them to reduce their emissions by large amounts. This stands to reason because such countries benefit little from climate change mitigation but have to bear costs of reducing their emissions that are higher than their benefits. Example 1 below illustrates this fact. Since in reality countries are sovereign and highly asymmetric, it follows that some countries may not be willing to participate in games whose outcomes can be supported as history dependent subgame perfect equilibria through the use of trigger strategies. In contrast, subgame-perfect cooperative agreements use transfers to balance the costs and benefits of controlling climate change and induce the countries to voluntarily participate in the agreement.

Example 1

Let $T = 2$, $N = \{1, 2\}$, $g_i(e_{it}) = 2e_{it}^{\frac{1}{2}}, i = 1,2, d_1(z_t) = \frac{1}{2}z_t$, , and $d_2(z_t) = z_t$, $z_0 = 0$, $\beta = 1$, and $\delta = 0$. Notice that the marginal damages of country 1 are only half of those of country 2.

In view of (2) and (3), the unique efficient emissions time-profile is given by $e_{i1}^* = \frac{1}{9}, e_{i2}^* = \frac{4}{9}, i = 1,2$. Thus $z_1^* = e_{11}^* + e_{21}^* = \frac{2}{9}, z_2^* = z_1^* + e_{12}^* + e_{22}^* = \frac{2}{9} + \frac{8}{9} = \frac{10}{9}$. Using backward induction, the unique SPNE strategies are given by $\bar{e}_{11} = 1, \bar{e}_{21} = \frac{1}{4}, \bar{e}_{12} = 4$, and $\bar{e}_{22} = 1$. Therefore, $\bar{z}_1 = \bar{e}_{11} + \bar{e}_{21} = 1 + \frac{1}{4} = \frac{5}{4}, \bar{z}_2 = \bar{z}_1 + \bar{e}_{12} + \bar{e}_{22} = \frac{5}{4} + 4 + 1 = \frac{25}{4}$. Thus, compared to the BAU emissions, efficiency requires country 1 to reduce its emissions in both periods by higher amounts than country 2: $\bar{e}_{11} - e_{11}^* = 1 - \frac{1}{9} > \bar{e}_{21} - e_{21}^* = \frac{1}{4} - \frac{1}{9}$ and $\bar{e}_{12} - e_{12}^* = 4 - \frac{4}{9} > \bar{e}_{22} - e_{22}^* = 1 - \frac{4}{9}$, though its benefits from efficient control of climate change are only half as much as those of country 2. The question is: will both countries be better-off if they indeed reduce their emissions to efficient levels?

Using the computations above, the payoffs of countries 1 and 2 if they reduce their emissions to efficient levels are $W_1^* = 2(e_{11}^*)^{\frac{1}{2}} - \frac{1}{2}z_1^* + 2(e_{12}^*)^{\frac{1}{2}} - \frac{1}{2}z_2^* = \frac{2}{3} - \frac{1}{9} + \frac{4}{3} - \frac{5}{9} = \frac{4}{3}$ and $W_2^* = 2(e_{21}^*)^{\frac{1}{2}} - z_1^* + 2(e_{22}^*)^{\frac{1}{2}} - z_2^* = \frac{2}{3} - \frac{2}{9} + \frac{4}{3} - \frac{10}{9} = \frac{2}{3}$, respectively, whereas their SPNE/BAU payoffs are $\bar{W}_1 = 2(\bar{e}_{11})^{\frac{1}{2}} - \frac{1}{2}\bar{z}_1 + 2(\bar{e}_{12})^{\frac{1}{2}} - \frac{1}{2}\bar{z}_2 = 2 - \frac{5}{8} + 4 - \frac{25}{8} = \frac{9}{4}$ and $\bar{W}_2 = 2(\bar{e}_{21})^{\frac{1}{2}} - \bar{z}_1 + 2(\bar{e}_{22})^{\frac{1}{2}} - \bar{z}_2 = 1 - \frac{5}{4} + 2 - \frac{25}{4} = -\frac{9}{2}$. This shows that $W_1^* < \bar{W}_1$. Thus, country 1 will not participate in any agreement for efficient control of climate change unless it is given transfers to compensate it for the resulting loss in its welfare. But is it possible to make transfers such that, after transfers, both countries will be better-off.

Since $W_1^* + W_2^* = 2 > \bar{W}_1 + \bar{W}_2 = -\frac{9}{4}$, there indeed exist transfers s_1 and s_2 such that $s_1 + s_2 = 0$ and $W_1^* + s_1 \geq \bar{W}_1$ and $W_2^* + s_2 \geq \bar{W}_2$. For example, if $s_1 = \frac{7}{3}$ and $s_2 = -\frac{7}{3}$, then $W_1^* + s_1 = \frac{4}{3} + \frac{7}{3} \geq \bar{W}_1 = \frac{9}{4}$ and $W_2^* + s_2 = \frac{2}{3} - \frac{7}{3} \geq \bar{W}_2 = -\frac{9}{2}$. A further question is : what should be the time-profile of these transfers? Should the transfers be such that they make both countries better-off in each period rather than just over the entire duration of the agreement? We pursue this question in the next section.

Finally, it may be noted that if the countries were symmetric, then $W_1^* = W_2^* > \bar{W}_1 = \bar{W}_2$, i.e. no transfers are necessary for the countries to be better-off if they both reduce their emissions to efficient levels.

4. Subgame-perfect agreements

As in Dutta and Radner (2009), we interpret the SPNE, especially since it is also Markov perfect, as the BAU equilibrium and the SPNE payoff of a country as the payoff that the country can assure for itself without cooperation of the other countries. However, since, for reasons mentioned above, we also consider coalitional behavior, we need to specify the payoff that a non-singleton coalition can similarly assure for itself without cooperation of the other countries. To that end, given the dynamic game Γ_{z_0}, for each coalition $S \subset N$, let $\Gamma_{z_0}^S$ denote the induced dynamic game in which coalition S acts as one single player, that is, within the coalition the individual strategies are selected so as to maximize the sum of the payoffs of its members, given the strategies of the non-members. Similarly, let $\Gamma^{S}_{z_{t-1}}$ denote an induced game of the subgame $\Gamma_{z_{t-1}}$, to be called an induced subgame. The induced games $\Gamma^{S}_{z_0}$ and $\Gamma^{S}_{z_{t-1}}$ may seem to have the same structures as the original games Γ_{z_0} and $\Gamma_{z_{t-1}}$, respectively, except that the number of players is $n - |S| + 1$ instead of n. But there is an important difference in that, unlike the original game Γ_{z_0}, the payoff of one of the players: namely coalition S, is a function of as many variables as the number of members of S. Therefore, existence and characterization of a SPNE for the induced games $\Gamma^{S}_{z_0}, S \subset N$, do not follow simply from Theorem 1 and have to be established, except in two cases of coalition structures in that the unique SPNE of Γ_{z_0} is also a unique SPNE of each induced game $\Gamma^{\{i\}}_{z_0}, i = 1, \ldots, n$, and the efficient emission time-profile $(e^*_{1t}, \ldots, e^*_{nt})^{T}_{t=1}$ is the unique SPNE of the induced game $\Gamma^{N}_{z_0}$.

Theorem 2 For each coalition $S \subset N$, each induced subgame $\Gamma^{S}_{z_{t-1}}, z_{t-1} \geq 0, t = 1, \ldots, T$, admits a unique subgame-perfect Nash equilibrium if the benefit functions $g_i, i = 1, \ldots n$ are strictly concave, the damage functions $d_i, i = 1, \ldots, n$ are strictly convex or linear, and the third derivatives $g_i''' = d_i''' = 0, i = 1, \ldots n$.

The proof of this theorem is also by backward induction, though, unlike in the proof for the existence of a unique SPNE, the payoff of one of the players: namely coalition S, in each induced subgame $\Gamma^{S}_{z_{t-1}}, z_{t-1} \geq 0, t = 1, \ldots, T$ is, as also noted above, a function of as many variables as the number of members of S. The method of backward induction works in this case also because the damage functions of all countries (though not necessarily identical) are functions of the same variables and, as seen from a comparison of equations (6) and (12)-(13) in the Appendix, the conditions characterizing the Nash equilibrium in both cases have the same mathematical structure.

Corollary 2.1 For each coalition S and the induced subgame $\Gamma^{S}_{z_{t-1}}$, the subgame-perfect Nash equilibrium strategy of each player, including each individual player in coalition S, is linear and non-increasing in the state variables $z_{t-1}, t = 1, \dots, T$.

The significance of this corollary is that the equilibrium strategy of each individual player, including each *individual* player in coalition S, is a Markov strategy. This seems to extend applications of Markov strategies even to concepts of cooperation in dynamic games. The proof for this corollary follows from the proof of Theorem 2 which shows that $e'_{it}(z_{t-1}) \leq 0$ and $e''_{it}(z_{T-1}) = 0$ irrespective of whether player i is a member of coalition S or not.

Corollary 2.2 The unique subgame-perfect Nash equilibrium payoff $w(S, z_{t-1})$ of coalition S is a non-increasing and concave function of z_{t-1}.

The definition of $w(S, z_{t-1})$ involves two assumptions. First, it does not require coalition S to cooperate with the countries outside S in any future period. Since the countries are sovereign and coalition S is free to not cooperate with the outside countries now or in the future, $w(S, z_{t-1})$ is among the possible payoffs that coalition S can achieve for itself without cooperation of the other countries. This means a cooperative agreement cannot be stable unless it promises each coalition S a payoff of at least $w(S, z_{t-1})$ in the subgame $\Gamma_{z_{t-1}}$. Second, the definition of $w(S, z_{t-1})$ assumes that the countries outside the deviating coalition S choose their *individually* best reply strategies rather than their *joint* best reply strategies.[12] This assumption is also implicit in Dutta and Radner (2009) who, however, restrict deviations to singleton coalitions only and assume that a deviation by a single country will result in reversal to the MPE, which they refer to as the BAU equilibrium and that, as shown above, is also the SPNE. Thus, in their formulation as well in this paper if a single country deviates then its payoff is equal to its MPE/SPNE payoff, i.e., $w(\{i\}, z_{t-1}) = q_i(z_{t-1})$. But if we assume instead that the remaining countries form a coalition of their own and choose their joint best reply strategies – which is as arbitrary as assuming that the remaining countries form singletons and choose their individually best reply strategies – then the payoff of a deviating singleton country will not be equal to its MPE/SPNE payoff and our approach would no longer be consistent with the concept of a MPE/SPNE. In fact, it would lead to a different concept in which a deviation by a single country will not result – except in the case of only two countries – in reversion to the BAU equilibrium, but to an equilibrium in which the deviating country plays its best reply strategy against the joint best reply strategies of the other countries.

Responding to a deviation by a coalition with individually best reply strategies can also be interpreted as imposing a mild punishment on the deviating coalition. It is mild because the remaining countries can impose instead a harsher punishment on the deviating

[12] This assumption has a long tradition in game theory: See e.g. Chander and Tulkens (1997), Helm (2000, 2003), Chander (2007), and Rubio and Ulph (2007) among others in the context of climate change games, and Rajan (1989), Lardon (2012), and Stamatopoulos (2016) among others in the context of an oligopoly.

coalition by choosing their highest feasible emissions in the current and all future periods, i.e., $e_{i\tau} = e^0$ for each $i \notin S$ and $\tau \geq t$, where t is the period in which the deviation occurs.[13] Instead, the remaining players are assumed to respond to the deviation as if each one of them and the deviating coalition have decided to go their separate ways in pursuit of maximizing their own payoffs. If the deviating coalition suffers any loss in its payoff, it is incidental and not the intention of the remaining players. Thus, it amounts to non-cooperation, but not to declaring a war or imposing the harshest punishment possible on the deviating coalition.

Definition 2 A subgame-perfect cooperative agreement in the dynamic game Γ_{z_0} is a consumption time-profile $(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^{T}$ such that for each coalition $S \subset N$ and each subgame $\Gamma_{z_{t-1}}$, $w(S, z_{t-1}) \leq \sum_{i \in S} \sum_{\tau=t}^{T} \beta^{\tau-t}(x_{i\tau} - d_i(z_\tau))$ for each, $t = 1, \dots T$.

The definition requires that the transfers between countries, implicit in the variables $x_{it}, i = 1, \dots, n; t = 1, \dots, T$, should be such that no country or coalition of countries is worse-off in *any* subgame. Since the payoffs $w(S, z_{t-1}), S \subset N$, depend on the state variable z_{t-1} each x_{it} also depends on the state variable z_{t-1}, though, to save on notation, this is not explicitly indicated in the definition. Furthermore, as will also become clear below, the transfers in any subgame $\Gamma_{z_{t-1}}$ do not depend on transfers in other subgames. This is because if a coalition deviates in some subgame then, by definition, it deviates forever and, therefore, its' – before or after transfers – payoffs in other subgames are not relevant.

[13] This harsher punishment is conceptually equivalent to the punishment in Marx and Matthews (2000) that the remaining players contribute nothing to the public good once a player deviates because that is the worst the remaining players can do in their model.

Indeed, since the unique efficient emissions time-profile $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ is also the unique SPNE of the "one-player" induced game $\Gamma_{z_0}^N$, we have $w(N; z_0) = \sum_{t=1}^T \beta^{t-1} \sum_{i \in N}[g_i(e_{it}^*) - d_i(z_t^*)]$, where $(z_t^*)_{t=1}^T$ is generated by the unique efficient emissions time-profile $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ according to Eq. (2). This means that only an efficient consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$, where $\sum_{i \in N} x_{it}^* = \sum_{i \in N} g_i(e_{it}^*)$, $t = 1, \dots, T$, can be a subgame-perfect cooperative agreement. Thus, the search for a subgame-perfect cooperative agreement is restricted to the set of efficient consumption time-profiles and the set of subgame-perfect cooperative agreements is a refinement of the set of efficient consumption time-profiles. Furthermore, an efficient consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ is a subgame-perfect cooperative agreement if and only if each restriction $(x_{1\tau}^*, \dots, x_{n\tau}^*; z_\tau^*)_{\tau=t}^T$, $t = 1, \dots, T$, of the efficient consumption time-profile belongs to the core of the characteristic function game $(N, w(., z_{\tau-1}^*))$, where $w(., z_{\tau-1}^*)$ denotes the characteristic function $w(S, z_{\tau-1}^*), S \subset N$. Since $w(N, z_{t-1}^*) = \sum_{\tau=t}^T \beta^{\tau-t} \sum_{i \in N}[g_i(e_{i\tau}^*) - d_i(z_\tau^*)]$, the dynamic game Γ_{z_0} admits a subgame-perfect cooperative agreement if and only if not only each of the T characteristic function games $\left(N, w(., z_{\tau-1}^*)\right), \tau = 1, \dots, T$, is balanced, but also appropriate restrictions of the *same* efficient consumption time-profile belong to the core of each of these T characteristic function games.

Definition 2 implies that an efficient consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ satisfies subgame perfection, if each restricted consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=\tau}^T$ satisfies subgame perfection in the subgame $\Gamma_{z_{\tau-1}^*}, \tau = 1, \dots, T$.[14] To understand why an agreement

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[14]It is worth noting in this connection that if a consumption time-profile $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ is efficient in the game Γ_{z_0}, then the restricted consumption time-profile $(x_{1\tau}^*, \dots, x_{n\tau}^*; z_\tau^*)_{\tau=t}^T$ is efficient in the subgame $\Gamma_{z_{\tau-1}^*}, \tau = 1,2, \dots, T$.

should have this property, consider an efficient consumption time-profile $(x'_{1t}, \dots, x'_{nt}; z^*_t)_{t=1}^{T}$ such that $\sum_{i\in S}\sum_{t=1}^{T}\beta^{t-1}\left(x'_{it} - d_i(z^*_t)\right) \geq w(S, z_0)$ for all $S \subset N$, but for some $S \subset N$ and period $\tau > 1$, $\sum_{i\in S}\sum_{t=\tau}^{T}\beta^{t-\tau}(x'_{it} - d_i(z^*_t) < w(S, z^*_{\tau-1})$. In words, consumption time-profile $(x'_{1t}, \dots, x'_{nt}; z^*_t)_{t=1}^{T}$ as an agreement is such that no coalition has incentive to withdraw from it in period 1, but some coalition S has incentive to withdraw from it at a future date τ and force renegotiations of the terms of the agreement – in contradiction to the beliefs of the other countries in period 1 that S will do its part when the time comes. We confirm this possibility by building on Example 1.

Example 2

Consider again the game in Example 1 with $\beta = 1$, and $\delta = 0$. Using the computations for the unique efficient emissions time-profile in Example 1, the set of all efficient consumption time-profiles $((x_{11}, x_{21}; z_1), (x_{12}, x_{22}; z_2))$ is described by the equalities $x_{11} + x_{21} = \frac{4}{3}$, $x_{12} + x_{22} = \frac{8}{3}$, $z^*_1 = \frac{2}{9}$, and $z^*_2 = \frac{10}{9}$. Thus, $w(\{1,2\}, z_0) = x_{11} + x_{21} - \frac{1}{2}z^*_1 - z^*_1 + x_{12} + x_{22} - \frac{1}{2}z^*_2 - z^*_2 = \frac{4}{3} - \frac{1}{9} - \frac{2}{9} + \frac{8}{3} - \frac{5}{9} - \frac{10}{9} = 2$, and $w(\{1,2\}, z^*_1) = x_{12} + x_{22} - \frac{1}{2}z^*_2 - z^*_2 = \frac{8}{3} - \frac{5}{9} - \frac{10}{9} = 1$. Next, using the computations for the unique SPNE in Example 1, we have $w(\{1\}, z_0) = 2(\bar{e}_{11})^{\frac{1}{2}} - \frac{1}{2}\bar{z}_1 + 2(\bar{e}_{12})^{\frac{1}{2}} - \frac{1}{2}\bar{z}_2 = 2 - \frac{5}{8} + 4 - \frac{25}{8} = \frac{9}{4}$, $w(\{2\}, z_0) = 2(\bar{e}_{21})^{\frac{1}{2}} - \bar{z}_1 + 2(\bar{e}_{22})^{\frac{1}{2}} - \bar{z}_2 = 1 - \frac{5}{4} + 2 - \frac{25}{4} = -\frac{9}{2}$, $w(\{1\}, z^*_1) = 2(\bar{e}_{12})^{\frac{1}{2}} - \frac{1}{2}(z^*_1 + \bar{e}_{12} + \bar{e}_{22}) = 4 - \frac{1}{2}(\frac{2}{9} + 5) = \frac{25}{18}$, and $w(\{2\}, z^*_1) = 2(\bar{e}_{22})^{\frac{1}{2}} - (z^*_1 + \bar{e}_{12} + \bar{e}_{22}) = 2 - (\frac{2}{9} + 5) = -\frac{29}{9}$.

Clearly, $\big((x_{11}, x_{21}; z_1), (x_{12}, x_{22}; z_2)\big) = ((-\frac{5}{3}, 3, \frac{2}{9}), (6, -\frac{10}{3}, \frac{10}{9}))$ is an efficient consumption time-profile and requires country 1 to transfer $\frac{7}{3}$ units of the private good to country 2 in period 1 and country 2 to transfer $\frac{14}{3}$ units to country 1 in period 2 and generates total payoffs of $r_{11} = x_{11} - \frac{1}{2}z_1^* + x_{12} - \frac{1}{2}z_2^* = -\frac{5}{3} - \frac{1}{9} + 6 - \frac{5}{9} = \frac{11}{3} > \frac{9}{4} = w(\{1\}, z_0)$ for country1 and $r_{21} = x_{12} - z_1^* + x_{22} - z_2^* = 3 - \frac{2}{9} - \frac{10}{3} - \frac{10}{9} = -\frac{5}{3} > -\frac{9}{2} = w(\{2\}, z_0)$ for country 2. Furthermore, $r_{11} + r_{21} = 2 = w(\{1,2\}, z_0)$. Thus, no country or coalition of countries can be better off by withdrawing in period 1 if the agreement is the efficient consumption time-profile $((-\frac{5}{3}, 3, \frac{2}{9}), (6, -\frac{10}{3}, \frac{10}{9}))$. But for the restricted consumption time-profile $(x_{12}, x_{22}; z_2) = (6, -\frac{10}{3}, \frac{10}{9})$ in period 2, $w(\{2\}, z_1^*) = -\frac{29}{9} > -\frac{10}{3} - \frac{10}{9} = -\frac{40}{9}$, and, therefore, country 2 will be better-off if it withdraws from the agreement in period 2, but not if it withdraws in period 1.

Country 1 would adhere to the agreement in period 1 because it expects to receive a large enough transfer in period 2, though it has to make a transfer in period 1 and country 2 would adhere with it in period 1 because it is to receive a large enough transfer in period 1, though it has to make a transfer in period 2. But come period 2 and state z_1^* (after country 1 has reduced its emissions in period 1 and made transfers) country 2 realizes that it would be better-off if it now leaves the agreement rather than comply with it. In fact, country 2 can force renegotiation of the terms of the agreement after the game reaches period 2, but not before period 2. For instance, it can abandon the current agreement and then propose instead the consumption profile $\left(4, -\frac{4}{3}, \frac{10}{9}\right)$ as the new agreement in period 2 which can make both the countries better off, but offers only 4 units of the private good to country 1 compared to 6 units in the original agreement.

The example explains why Definition 2 requires that no country or coalition of countries should have incentive to leave the agreement in any period and force renegotiations of the terms of the agreement. It can be easily verified that if the agreed upon efficient consumption time-profile were instead $((\frac{1}{3}, 1, \frac{2}{9}), \left(4, -\frac{4}{3}, \frac{10}{9}\right))$, then no country will have incentive to withdraw from the agreement in any period.

4.2 *Existence and further characterization*

We now identify sufficient conditions for the existence of a subgame-perfect cooperative agreement for the dynamic game Γ_{z_0}.

Theorem 3 The dynamic game Γ_{z_0} admits a subgame-perfect cooperative agreement $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ if the benefit functions g_i are strictly concave with $g_i''' = 0$, $i = 1, \dots n$, and the damage functions $d_i, i = 1, \dots, n$ are linear.

The proof of this theorem shows that if the damage functions are linear, then the transfers required by a subgame-perfect cooperative agreement, like the SPNE/MPE strategies, do not depend on the state variable. In fact, the dynamic game can be decomposed into a sequence of strategic games that do not depend on the state variable and that can be solved independently of each other. But to meet the requirement of subgame perfection, their γ-core (see fn.15 below for definition) solutions must be combined such that no coalition will have incentive to deviate in any period. For instance, if some coalition is given “too high” transfers in period $T - 1$, then it will have to make “too high” transfers in period T and thus it will have incentive to withdraw from the agreement in period T, though not in period $T-1$.

Subgame-perfect cooperative agreements also exist if the damage functions are strictly convex, but the countries are symmetric. In this case no transfers are necessary and $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ where $x_{it}^* = g_i(e_{it}^*), i = 1, \dots, n; t = 1, \dots, T$, is a subgame-perfect cooperative agreement. But if the damage functions are strictly convex and the countries are not symmetric, proving existence of a subgame-perfect cooperative agreement is technically far more challenging, as the dynamic game then cannot be decomposed into a sequence of strategic games that are independent of the state variable and thus can be solved independently of each other. However, since proving existence in this more general case will not lead to additional insights regarding subgame-perfect cooperative agreements and at best only of technical interest, we leave it as a future research project.

4.3 *Infinite time horizon*

Theorem 3 also holds for $T = \infty$ and $0 < \beta(1-\delta) < 1$. In fact, if the time horizon $T = \infty$, a subgame-perfect cooperative agreement is constructed as follows. As in the proof of Theorem 3, the dynamic game can be decomposed into (infinitely many) strategic games $\Omega_{z_{t-1}^*}, t = 1,2, \dots$ with $g_i(e_{it}) - c_i \sum_{\tau=t}^{\infty} [\beta(1-\delta)]^{\tau-t} \left(\sum_{j \in N} e_{jt} + (1-\delta) z_{t-1}^*\right) = g_i(e_{it}) - \frac{c_i}{1-\beta(1-\delta)} \left(\sum_{j \in N} e_{jt} + (1-\delta) z_{t-1}^*\right)$, since $0 < \beta(1-\delta) < 1$, as the payoff function of player i in the strategic game $\Omega_{z_{t-1}^*}$. This payoff function is strictly concave in e_{it}, since g_i is strictly concave. Thus, each strategic game $\Omega_{z^*_{t-1}}$ admits a unique Nash equilibrium. This Nash equilibrium is independent of the state variable z^*_{t-1} and characterized by equalities (9) in the Appendix. This implies that the dynamic game with infinite time horizon and linear damage functions admits a unique SPNE in stationary strategies. Let $(\bar{e}_1, \ldots, \bar{e}_n)$ denote the stationary SPNE strategies. Similarly, let $(e^*_1, \ldots, e^*_n)$ denote the stationary efficient emissions as characterized by (4) above.

Let $w_t(S), S \subset N, t = 1,2, \ldots$ denote the unique Nash equilibrium payoff of coalition S in the induced game of the strategic game $\Omega_{z^*_{t-1}}$ in which coalition S acts as one player and the remaining players act as singletons. Since the payoff functions are strictly concave and the damage functions are linear, by Theorem 1 in Chander and Tulkens (1997), the core[15] of each characteristic function game $(N, w_t), t = 1,2, \ldots$ is nonempty and the imputation

$$r_{it} = x^*_i - \frac{c_i}{1-\beta(1-\delta)}\left(\sum_{j\in N} e^*_j + (1-\delta)\, z^*_{t-1}\right), i = 1, \ldots, n, \tag{14}$$

where $x^*_i = g_i(\bar{e}_i) - \frac{c_i}{\sum_{j\in N} c_j}\sum_{j\in N}[g_j\left(\bar{e}_j\right) - g_j(e^*_j)]$ belongs to the core, i.e. $w_t(S) \le \sum_{i\in S} r_{it}$.

Since the Nash equilibrium of each induced game of each strategic game $\Omega_{z^*_{t-1}}$, is independent of the stock z^*_{t-1}, we have $w(S, z^*_{t-1}) = \sum_{t=1}^{\infty} \beta^{t-1} w_t(S)$. Since $w_t(S) \le \sum_{i\in S} r_{it}$, we have $w(S, z^*_{t-1}) \le \sum_{t=1}^{\infty} \beta^{t-1} w_t(S) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_{i\in S}[x^*_i - \frac{c_i}{1-\beta(1-\delta)}\left(\sum_{j\in N} e^*_j + (1-\delta)\, z^*_{t-1}\right)]$, by (14) above. This means the efficient consumption time-profile

[15] It is called the γ-core in Chander and Tulkens (1997). Also, see Helm (2000).

$(x_1^*, \ldots, x_n^*; z_t^*)_{t=1}^{\infty}$ with $z_t^* = (1-\delta)z_{t-1}^* + \sum_{j \in N} e_j^*$ is a subgame-perfect cooperative agreement.

Notice that the above arguments are analogous to those in the proof of Theorem 3 in that they also exploit the fact that the damage functions are linear and, therefore, both the SPNE strategies and the efficient emission time-profile in each subgame are independent of the GHG stock. However, since, unlike the dynamic game with finite horizon, each subgame of the dynamic game with infinite horizon has exactly the same structure, the SPNE strategies and the efficient emissions are stationary and the same transfers in each period ensure a subgame-perfect cooperative agreement.

5. Concluding remarks

The model and analysis in this paper are driven by two most important aspects of the climate change problem: The marginal abatement costs are decreasing in emissions levels and differ across the countries and the countries are sovereign and highly asymmetric. In contrast, most previous studies either assume identical and constant marginal abatement costs and/or sufficiently symmetric countries. Our analysis showed that if the countries are not sufficiently symmetric then transfers between them to balance the costs and benefits of controlling climate change are not a matter of approach, but a necessity.[16] These transfers should not only make each country or coalition of countries better-off over the entire duration of the agreement but for an

[16] Typically, as seen in Example 1, countries with relatively low marginal damages will not be willing to voluntarily participate in an agreement that requires it to reduce its emissions by largeamounts but does not compensate them adequately for reducing their emissions.

agreement to be stable during its entire duration the transfers should also have this property in every subgame. In contrast, history dependent subgame perfect equilibria that can be sustained through trigger strategies are either inefficient or not a Pareto improvement over the BAU equilibrium, since, by definition, they rule out transfers to balance the costs and benefits of controlling climate change.

By their appearances, the subgame-perfect cooperative agreements proposed in this paper may look quite different from actual climate agreements, since unlike the Kyoto Protocol and the Paris Agreement they require direct transfers between countries. But this is not really so. Since the Kyoto Protocol made provision for international trade in emissions, it was really an agreement on emission quotas that could be traded on an international market and that after (competitive) trade in emissions would have resulted in transfers that are similar to those in a subgame-perfect cooperative agreement. Conversely, since a subgame-perfect cooperative agreement, by definition, is an efficient consumption time-profile, it is, as shown in Section 2.1, equivalent to an agreement on a time-profile of internationally tradable emission quotas that does not require direct transfers between countries, but that after trade in quotas would result in exactly the same transfers. Since the Paris Agreement, like the Kyoto Protocol, also provides for international trade in emissions, the intended nationally determined contributions (INDCs) chosen voluntarily by the signatories to the Agreement can be interpreted as the agreed upon internationally tradable emission quotas. It is an open question whether some countries will have incentives to comply with the Paris Agreement at the beginning, but withdraw from it in a later period when it will be their turn to carry out deeper cuts in their emissions.

Our model is consistent with two actual agreements: The Kyoto Protocol and the more recent Paris Agreement. The Kyoto Protocol was ratified by only a subset of 36 countries. But, as assumed in the present paper, the countries which did not ratify the Protocol, including the US, remained singletons and did not form a coalition of their own. Given the non-ratification of the Kyoto Protocol by the US – the biggest polluter then – the ratifying countries realized the futility of implementing the Protocol[17] and soon abandoned it in favor of the Paris Agreement that has been signed by as many as 196 countries: the coalition of almost all countries!

Future research should address a number of simplifying assumptions that were made in order to highlight the strategic aspects of the climate change problem. We assumed no capital accumulation and no technological progress.[18]

Appendix

Proof of Lemma 1: Equalities (2) and (3) form a system of $(n+1)T$ equations in $(n+1)T$ variables and, therefore, admit a solution. Let $(e_{1t}^{*} \dots, e_{nt}^{*})_{t=1}^{T}$, $(z_t^{*})_{t=1}^{T}$ be a solution. We claim that it is unique: Suppose contrary to the assertion that the equalities admit two solutions. Then, since the objective function W is strictly concave and constraints (2) are linear, a convex combination of the two solutions also satisfies constraints (2) and implies a higher value of W

[17]To be precise, a second commitment period was agreed upon in 2012, known as the Doha Amendment to the Kyoto Protocol. But only a small subset of countries accepted the Doha Amendment. However, it may be argued that the US was the largest emitter and thus a special case. But if a country is small emitter, then it does not matter a great deal whether it is a member of the grand coalition or not, since its equilibrium strategy/emissions would more or less be the same. In other words, a coalition that includes all big emitters, but leaves out some small emitters is effectively the grand coalition. To put it differently, *only* a coalition that does not include some large emitter is effectively not the grand coalition.

[18] See Hong and Karp (2012), Dutta and Radner (2004), and Harstad (2012) for static and dynamic models of technical progress without transfers between countries.

contradicting the supposition that both solutions maximize W. Hence, the solution

$(e^*_{1t}, \dots, e^*_{nt})^T_{t=1}$, $(z^*_t)^{\,T}_{t=1}$ is unique. ∎

Proof of Theorem 1: To keep the algebra simple, we prove the theorem for $\beta = 1$, and $\delta = 0$. The proof for the more general case $\beta \leq 1$ and $\delta \geq 0$ is analogous.

We show that backward induction leads to a unique SPNE. Begin with a subgame in the last period T. A strategy profile $(e_{1T}, \dots, e_{nT})$ is a SPNE of a last period subgame $\Gamma_{z_{T-1}}$if each e_{iT} maximizes $g_i(e_{iT}) - d_i\left(z_{T-1} + \sum_{j \in N} e_{jT}\right)$, given $e_{jT}, j \neq i$. Therefore, the first order conditions (FOCs) for payoff maximization imply

$$g_i'(e_{iT}) = d_i'\left(z_{T-1} + \textstyle\sum_{j \in N} e_{jT}\right), i = 1, \dots, n. \tag{6}$$

We claim that these equations have a unique solution. Suppose not, and let $(\bar{e}_{1T}, \dots, \bar{e}_{nT})$ and $(\bar{\bar{e}}_{1T}, \dots \bar{\bar{e}}_{nT})$ be two different solutions such that $\sum_{i \in N} \bar{e}_{iT} = (>) \sum_{i \in N} \bar{\bar{e}}_{iT}$. Then, since each d_i is convex and g_i is strictly concave, (6) implies $\bar{e}_{iT} = (<) \bar{\bar{e}}_{iT}$ for $i = 1, \dots, n$, which contradicts our supposition. Hence, the last period subgame $\Gamma_{z_{T-1}}$ admits a unique SPNE for $z_{T-1} \geq 0$. Let $(e_{1T}(z_{T-1}), \dots, e_{nT}(z_{T-1}))$ denote the unique SPNE of $\Gamma_{z_{T-1}}$. By differentiating (6),

$$g_i''\left(e_{iT}(z_{T-1})\right)e_{iT}'(z_{T-1}) = d_i''(z_{T-1} + \textstyle\sum_{j \in N} e_{jT}(z_{T-1}))(1 + \sum_{j \in N} e_{jT}'(z_{T-1}))\,, i \in N. \tag{7}$$

Since $g_i'' < 0$ and $d_i'' \geq 0$, equations (7) imply $e_{iT}'(z_{T-1}) \leq 0$ and $(1 + \sum_{j \in N} e_{jT}'(z_{T-1}) \geq 0$. By differentiating (7), $g_i'''\left(e_{iT}(z_{T-1})\right)(e_{iT}'(z_{T-1}))^2 + g_i''\left(e_{iT}(z_{T-1})\right)e_{iT}''(z_{T-1}) =$

$d_i'''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}')^2 + d_i''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1})) \sum_{j\in N} e_{jT}''(z_{T-1})$,

$i = 1, \ldots, n$. Therefore,

$$g_i''\big(e_{iT}(z_{T-1})\big)e_{iT}''(z_{T-1}) = d_i''(z_{T-1} + \textstyle\sum_{j\in N} e_{jT}(z_{T-1})) \sum_{j\in N} e_{jT}''(z_{T-1}), i \in N, \qquad (8)$$

since $g_i''' = d_i''' = 0, i = 1, \ldots n$. Since $g_i'' < 0$ and $d_i'' \geq 0$, equations (8) imply $e_{iT}''(z_{T-1}) = 0$.

Let $q_i(z_{T-1}) \equiv g_i\big(e_{iT}(z_{T-1})\big) - d_i\big(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1})\big), i = 1, \ldots, n$. Then, $q_i'(z_{T-1}) = g_i'\big(e_{iT}(z_{T-1})\big)e_{iT}'(z_{T-1}) - d_i'(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}'(z_{T-1}) \leq 0$, since $g_i' > 0, d_i' > 0$, and, as shown, $e_{jT}'(z_{T-1}) \leq 0$ and $(1 + \sum_{j\in N} e_{jT}'(z_{T-1}) \geq 0$; and $q_i''(z_{T-1}) = g_i''\big(e_{iT}(z_{T-1})\big)(e_{iT}'(z_{T-1}))^2 - d_i''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}'(z_{T-1}))^2 + g_i'\big(e_{iT}(z_{T-1})\big)e_{iT}''(z_{T-1}) - d_i'(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1})) \sum_{j\in N} e_{jT}''(z_{T-1}) = g_i''\big(e_{iT}(z_{T-1})\big)(e_{iT}'(z_{T-1}))^2 - d_i''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}'(z_{T-1}))^2 \leq 0$, since $g_i'' < 0, d_i'' \geq 0$, and, as shown, $e_{iT}''(z_{T-1}) = 0, i = 1, \ldots, n$.

This proves that each $q_i(z_{T-1}), i = 1, \ldots, n$, is a non-increasing concave function of z_{T-1}. In fact, by differentiating the above expression and using $g_i''' = v_i''' = 0$ for each $i = 1, \ldots n$, it is seen that $q_i'''(z_{T-1}) = 0$. Thus, the reduced form of the subgame $\Gamma_{z_{T-2}}$ with payoff functions $g_i(e_{iT-1}) - [d_i\big(z_{T-2} + \sum_{j\in N} e_{jT-1}\big) - q_i\big(z_{T-2} + \sum_{j\in N} e_{jT-1}\big)]$ has exactly the same mathematical structure as the game $\Gamma_{z_{T-1}}$, since $q_i(z_{T-1}), i = 1, \ldots, n$, is a non-increasing concave and quadratic function of z_{T-1}. Therefore, $\Gamma_{z_{T-2}}$ admits a unique SPNE and the SPNE payoffs $q_i(z_{T-2}), i = 1, \ldots, n$, are similarly non-increasing concave and quadratic functions of

z_{T-2}. Continuing in this manner, the backward induction leads to a unique SPNE of the dynamic game Γ_{z_0}. ■

Proof of Proposition 2: For linear damage functions, the SPNE strategies are independent of the GHG stock and characterized by[19]

$$g_i'(e_{it}) = \frac{c_i}{1-\beta(1-\delta)}, i = 1, \dots, n; t = 1,2, \dots. \tag{9}$$

Let $(\bar{e}_1, \dots, \bar{e}_n)$ denote the solution of (9). Then, comparing (4) in subsection 2.2 and (9), the strict concavity of g_i implies $\bar{e}_i > e_i^*$, $i = 1, \dots, n$, and the SPNE payoff of country i is

$$\bar{W}_i = \frac{1}{1-\beta}[g_i(\bar{e}_i) - \frac{c_i \sum_{j=1}^n \bar{e}_j}{1-\beta(1-\delta)}] - \frac{c_i z_0}{1-\beta(1-\delta)}, i = 1, \dots, n. \tag{10}$$

Since the *unique* efficient GPO consumption time-profile, by definition, does not involve transfers between countries, the corresponding payoff of country i is

$$W_i^* = \frac{1}{1-\beta}\left[g_i(e_i^*) - \frac{c_i \sum_{j=1}^n e_j^*}{1-\beta(1-\delta)}\right] - \frac{c_i z_0}{1-\beta(1-\delta)} \tag{11}$$

where $(e_1^*, \dots, e_n^*)$ is the unique solution of (4) and, therefore, of (5) with equal welfare weights $\alpha_i, i = 1, \dots, n$. From (10) and (11), we obtain

$$\bar{W}_i - W_i^* = \frac{1}{1-\beta}[g_i(\bar{e}_i) - \frac{c_i \sum_{j=1}^n \bar{e}_j}{1-\beta(1-\delta)}] - \frac{1}{1-\beta}[g_i(e_i^*) - \frac{c_i \sum_{j=1}^n e_j^*}{1-\beta(1-\delta)}]$$

$$= \frac{1}{1-\beta}\left[g_i(\bar{e}_i) - g_i(e_i^*) - \frac{c_i}{1-\beta(1-\delta)}\left(\sum_{j=1}^n \bar{e}_j - \sum_{j=1}^n e_j^*\right)\right], i = 1, \dots, n.$$

[19] This can be proved in several ways. The simplest is perhaps to first show that the SPNE for T finite is characterized by the equations $g_i'(e_{it}) = \frac{c_i(1-(\beta(1-\delta))^{T-t+1})}{1-\beta(1-\delta)}, i = 1, \dots, n; t = 1, \dots T$, and then take the limit $T \to \infty$.

In this expression the terms $g_i(\bar{e}_i) - g_i(e_i^*) > 0$ and $\frac{c_i}{1-\beta(1-\delta)}\left(\sum_{j=1}^{n} \bar{e}_j - \sum_{j=1}^{n} e_j^*\right) > 0$, since, as shown, $\bar{e}_i > e_i^*$. As seen from (4) and (9), e_i^* is a decreasing function of $\sum_{j=1}^{n} c_j$ whereas $\bar{e}_i$ a decreasing function of c_i alone. Thus, for c_i sufficiently small but $\sum_{j=1}^{n} c_j$ sufficiently large with some $c_j > c_i, j \neq i$, we have $\overline{W}_i > W_i^*$. ■

Proof of Theorem 2: As in the case of Theorem 1, we need to prove the theorem only for $\beta = 1$ and $\delta = 0$. Thus, an emission profile $(e_{1T}, \dots, e_{nT})$ is a SPNE of an induced subgame $\Gamma_{z_{T-1}}^{S}$ in the last period T, if $(e_{iT})_{i \in S}$ maximizes $\sum_{i \in S} g_i(e_{iT}) - \sum_{i \in S} d_i(z_{T-1} + \sum_{j \in N} e_{jT})$ and each $e_{jT}, j \in N \backslash S$, maximizes $g_j(e_{jT}) - d_j(z_{T-1} + \sum_{i \in N} e_{iT})$. Therefore, by FOCs for payoff maximization, $(e_{1T}, \dots, e_{nT})$ must be a solution of the equations

$$g_i'(e_{iT}) = \sum_{j \in S} d_j'(z_{T-1} + \sum_{k \in N} e_{kT}), i \in S, \tag{12}$$

$$g_j'(e_{jT}) = d_j'(z_{T-1} + \sum_{i \in N} e_{iT}), j \in N \backslash S. \tag{13}$$

As in the proof of Theorem 1, these equations admit a unique solution $\left(e_{1T}(z_{T-1}), \dots, e_{nT}(z_{T-1})\right)$. Differentiating (12) and (13), we obtain

$$g_i''\left(e_{iT}(z_{T-1})\right)e_{iT}'(z_{T-1}) = \sum_{k \in S} d_k''(z_{T-1} + \sum_{j \in N} e_{jT}(z_{T-1}))\,(1 + \sum_{j \in N} e_{jT}'(z_{T-1})), i \in S.$$

$$g_j''\left(e_{jT}(z_{T-1})\right)e_{jT}'(z_{T-1}) = d_j''(z_{T-1} + \sum_{j \in N} e_{jT}(z_{T-1}))(1 + \sum_{j \in N} e_{jT}'(z_{T-1})), j \in N \backslash S.$$

Since each g_i is strictly concave and each d_i is convex, these equations imply $(1 + \sum_{j \in N} e_{jT}'(z_{T-1})) \geq 0$ and $e_{iT}'(z_{T-1}) \leq 0$. Differentiating these equations once more and using $g_i''' = d_i''' = 0$ for each i implies $e_{iT}''(z_{T-1}) = 0, i = 1, \dots, n$, and the SPNE payoffs are

$w(S, z_{T-1}) = \sum_{i\in S} g_i(e_{iT}(z_{T-1})) - \sum_{i\in S} d_i(z_{T-1} + \sum_{j\in N} e_{jT}\,(z_{T-1}))$ for coalition S and $m_j(z_{T-1}) = g_j(e_{jT}(z_{T-1})) - d_i(z_{T-1} + \sum_{j\in N} e_{jT}\,(z_{T-1}))$ for each $j \in N\backslash S$.

Differentiating $w(S, z_{T-1})$ and $m_j(z_{T-1})$ twice shows that each $w(S, z_{T-1})$, $S \subset N$, and $m_j(z_{T-1}), j = 1, \dots, n$ are non-increasing concave function of z_{T-1}. Furthermore, as in Theorem 1, $w'''(S, z_{T-1}) = m_j'''(z_{T-1}) = 0$. Thus, the reduced form of the induced subgame $\Gamma^S_{z_{T-2}}$ has exactly the same mathematical structure as the game $\Gamma^S_{z_{T-1}}$. Continuing in this manner, the backward induction leads to a unique SPNE of the induced subgame $\Gamma^S_{z_{t-1}}, t = 1, \dots, T$. ■

Proof of Theorem 3: To keep the algebra simple, we prove the theorem for $\beta = 1$, and $\delta = 0$. The proof for the more general case $\beta \leq 1$ and $\delta \geq 0$ is analogous. Let $d_i(z) = c_i z, i = 1, \dots, n$, be the damage functions.

We first prove the theorem for $T = 2$ and then extend the proof to any finite T. Since $\beta = 1$, and $\delta = 0$, the payoff of player i in period $T - 1$ is given by $g_i(e_{iT-1}) - c_i(z^*_{T-2} + \sum_{j\in N} e_{jT-1})$ $+g_i(e_{iT}) - c_i(z^*_{T-2} + \sum_{j\in N} e_{jT-1} + \sum_{j\in N} e_{jT}) = g_i(e_{iT-1}) - 2c_i \sum_{j\in N} e_{jT-1} + g_i(e_{iT}) - c_i \sum_{j\in N} e_{jT} - 2c_i z^*_{T-2}$. Since $T = 2$, $z^*_{T-2} = z_0 = 0$. Now consider two strategic form games, say Ω_{T-1} and Ω_T, in which the strategy sets are $E_{1t} \times E_{2t}$, where $E_{it} = \{(e_{i\tau})^T_{\tau=t}: 0 \leq e_{it} \leq e^0\}, i = 1,2; t = T-1, T$, respectively, and the payoff function of player $i \in \{1,2\}$ is $g_i(e_{iT-1}) - 2c_i \sum_{j\in N} e_{jT-1}$ in Ω_{T-1} and $g_i(e_{iT}) - c_i \sum_{j\in N} e_{jT}$ in Ω_T. The two games can be solved independently of each other. Since each g_i is strictly concave, let $w_{T-1}(S), S \subset N$, and $w_T(S), S \subset N$, denote the unique Nash equilibrium payoff of coalition S in the induced games of Ω_{T-1} and Ω_T, respectively, in which coalition S acts as one player and the remaining players act as singletons. By Theorem 1 in Chander and Tulkens (1997), the core ((i.e. the γ-core) of each characteristic function game (N, w_{T-1}) and (N, w_T) is nonempty. By definition of these strategic games, $w_{T-1}(N) = \sum_{i\in N}[g_i(e^*_{iT-1}) - 2c_i \sum_{j\in N} e^*_{jT-1}]$ and $w_T(N) = \sum_{i\in N}[g_i(e^*_{iT}) - c_i \sum_{j\in N} e^*_{jT}]$ where the emission time-profile $((e^*_{1T-1}, \dots, e^*_{nT-1}), (e^*_{1T}, \dots, e^*_{nT}))$ is the unique efficient emission time-profile in the (two-period) dynamic game and $z^*_{T-1} = z^*_{T-2} + \sum_{i\in N} e^*_{iT-1}$. Thus, there exist $(x^*_{1T-1}, \dots, x^*_{nT-1})$ and $(x^*_{1T}, \dots, x^*_{nT})$ such that $\sum_{i\in N} x^*_{iT-1} = \sum_{i\in N} g_i(e^*_{iT-1})$, $\sum_{i\in N} x^*_{iT-2} = \sum_{i\in N} g_i(e^*_{iT-2})$ and for each $S \subset N$, $w_{T-1}(S) \leq \sum_{i\in S}[x^*_{iT-1} - 2c_i \sum_{i\in S} e^*_{iT-1}]$ and $w_T(S) \leq \sum_{i\in S}[x^*_{iT} - c_i \sum_{i\in S} e^*_{iT}]$. By definition, $w(S, z^*_{T-2}) = w_{T-1}(S) + w_T(S) - 2\sum_{i\in S} c_i z^*_{T-2}$ and $w(S, z^*_{T-1}) = w_T(S) - c_i z^*_{T-1}$. Since $z^*_{T-1} = z^*_{T-2} + \sum_{i\in N} e^*_{iT-1}$, this implies that there exists a feasible consumption time-profile $(x^*_{1t}, \dots, x^*_{nt}; z^*_t)^T_{t=1}$ such that for each $S \subset N$, we have $w(S, z^*_{T-2}) \leq \sum_{i\in S}[x^*_{iT-1} - c_i(z^*_{T-2} + \sum_{j\in N} e^*_{jT-1})] + \sum_{i\in S}[x^*_{iT} - c_i(z^*_{T-1} + \sum_{j\in N} e^*_{jT}]$ and $w(S, z^*_{T-1}) \leq \sum_{i\in S}[x^*_{iT} - c_i(z^*_{T-1} + \sum_{j\in N} e^*_{jT}]$.

The proof for $T \geq 3$, is analogous and follows from the fact that if the damage functions are linear, then every T period dynamic game can be decomposed into T strategic games and a constant term. A consumption time-profile that satisfies subgame-perfection can be then constructed by combining a γ-core imputation of each of the strategic games. ■

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